$$\lim_{n\to\infty} \sqrt[n]{n! + n^n - \sin(n^{n!})\cdot n} \cdot \frac{\sqrt[6]{8n^3-9n+1} - \sqrt[10]{32n^5-60n^3+10n^2-4}}{\sqrt[6]{n^3+n} - \sqrt[6]{n^3+1}} =: \lim_{n\to\infty} a_n$$

Platí: $\sqrt[6]{8n^3-9n+1} - \sqrt[10]{32n^5-60n^3+10n^2-4} =$ $\quad \sqrt[6]{a} = \sqrt[30]{a^5}, \ \sqrt[10]{a} = \sqrt[30]{a^3}$

$$= \frac{(8n^3-9n+1)^5 - (32n^5-60n^3+10n^2-4)^3}{\sum_{k=0}^{29} \left(\sqrt[6]{8n^3-9n+1}\right)^{29-k} \left(\sqrt[10]{32n^5-60n^3+10n^2-4}\right)^k} =$$

$$= \frac{1}{n^{\frac{29}{2}} \sum_{k=0}^{29} \left(\left(8 - \frac{9}{n^2}+\frac{1}{n^3}\right)^{\frac{29-k}{6}} \left(32 - \frac{60}{n^2}+\frac{10}{n^3}-\frac{4}{n^5}\right)^{\frac{k}{10}}\right)} \cdot A, \text{ kde}$$

$$A = (8n^3+(-9n+1))^5 - (32n^5+(-60n^3+10n^2-4))^3 \overset{\text{binomická věta}}{=}$$

$$= \cancel{(8n^3)^5} + \binom{5}{1}(8n^3)^4(-9n+1) + P_{\le 10}(n) - \cancel{(32n^5)^3} - \binom{3}{1}(32n^5)^2(-60n^3+10n^2-4) - P_{\le 11}(n) =$$

$$= 5\cdot 8^4 \cdot n^{12}(-9n+1) - 3\cdot 32^2\cdot n^{10}(-60n^3+10n^2-4) + P_{\le 11}(n) =$$

$$= \cancel{n^{13}(-9\cdot 5\cdot 8^4)} + 5\cdot 8^4\cdot n^{12} + \cancel{n^{13}(3\cdot 32^2\cdot 60)} - 30\cdot 32^2 n^{12} + 12\cdot 32^2 n^{10} + P_{\le 11}(n) =$$

$$= n^{12}\big(\underbrace{5\cdot 8^4 - 30\cdot 32^2}_{=-10240}\big) + P_{\le 11}(n)$$

Tedy

$$\lim_{n\to\infty} a_n = \lim \sqrt[n]{n! + n^n - n\sin(n^{n!})} \cdot \frac{\sum_{k=0}^{5}\left(\sqrt[6]{n^3+n}\right)^{5-k}\left(\sqrt[6]{n^3+1}\right)^k}{n-1} \cdot \frac{n^{12} + P_{\le 11}(n)}{n^{\frac{29}{2}} \sum_{0}^{29} \left(8 - \frac{9}{n^2}+\frac{1}{n^3}\right)^{\frac{29-k}{6}} (\dots)^{\frac{k}{10}}} =$$

$$= \lim_{n\to\infty} \sqrt[n]{} \cdot \frac{1}{n} \cdot \underbrace{\text{výraz}}_{\text{konverguje k číslu}}, \quad \text{výraz} = \frac{\sum_0^5 \left(\sqrt[6]{1+\frac{1}{n^2}}\right)^{5-k}\left(\sqrt[6]{1+\frac{1}{n^3}}\right)}{\sum_0^{29} (8 - -)^{\frac{29-k}{6}} (32\ )^{\frac{k}{10}}}$$

a $\sqrt[n]{1-\frac{1}{n^{n-1}}} \le \frac{n}{n}\cdot \sqrt[n]{1+\frac{n!}{n^n} - \frac{1}{n^{n-1}}\sin(n^{n!})} \le \sqrt[n]{1+2} \longrightarrow 1$

$$\sqrt[x]{1-\frac{1}{x^{x-1}}} = e^{\frac{1}{x}\log\left(1-\frac{1}{x^{x-1}}\right)} \xrightarrow{\text{VOLSF+ZL}} e^0 = 1$$

Heine $\Rightarrow \sqrt[n]{1-\frac{1}{n^{n-1}}} \longrightarrow 1$

2 policajti $\Rightarrow \lim a_n = \lim \text{výrazu} \overset{\text{a spoj. VoAL}}{=} \frac{\sum_0^5 1}{\sum_0^{29} 8^{\frac{29-k}{6}}\cdot 32^{\frac{k}{10}}} = \frac{6}{2^{\frac{29}{2}}\cdot \sum_0^{29} 1} = \frac{6}{29\cdot 2^{\frac{29}{2}}}$